

Mathematics: applications and interpretation Standard level Paper 1

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Zone A afternoon | Zone B afternoon | Zone C afternoon

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1 hour 30 minutes

Instructions to candidates

- · Write your session number in the boxes above.
- · Do not open this examination paper until instructed to do so.
- · A graphic display calculator is required for this paper.
- Answer all questions.
- · Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the mathematics: applications and interpretation SL formula booklet is required for this paper.
- The maximum mark for this examination paper is [80 marks].



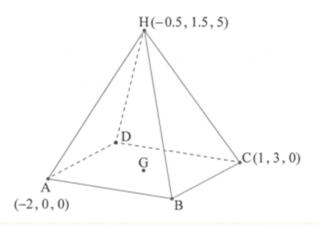
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

A metal structure on a flat surface is in the form of a right-pyramid with rectangular base ABCD and vertex H(-0.5, 1.5, 5). Point A has coordinates (-2, 0, 0) and point C has coordinates (1, 3, 0). This is shown in the following diagram.

All units are in centimetres.

diagram not to scale



The centre of the base, G, is the midpoint of AC.

(a) Find the coordinates of G. [2]

(b) Write down the vertical height HG. [1]

(c) Find the distance between C and H. [2]

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2. [Maximum mark: 5]

The monthly energy consumption, x, in kilowatt hours (kWh) of $150\,$ households in Helvetia is shown in the following table.

Energy consumption (x kWh)	Number of households
500 ≤ x < 700	38
700 ≤ <i>x</i> < 900	45
900 ≤ <i>x</i> < 1100	25
$1100 \le x < 1300$	18
$1300 \le x < 1500$	16
$1500 \le x < 1700$	8

(a)	For this data set.	use the mid-point	interval values	to find	estimates	for
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(i)	1 1	the	mean.
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(ii)	the	standa	ard c	leviat	ion.

[4]

The standard deviation of the monthly energy consumption in another nearby residential area, Eureka, is found to be $\,95\,kWh_{\,\cdot}$

(b)	Interpret the meaning of the value of the standard deviation in Eureka in comparison
	with the standard deviation in Helvetia.

[1]

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The water level, h, in metres, in a water tank after t hours of irrigation is modelled by the following function.

$$h(t) = \frac{20}{2t+5}, \ t \ge 0$$

(a) Find the value of h(0.5).

[2]

- (b) (i) Find the value of $h^{-1}(2.5)$.
 - (ii) Interpret this value in context.

[3]

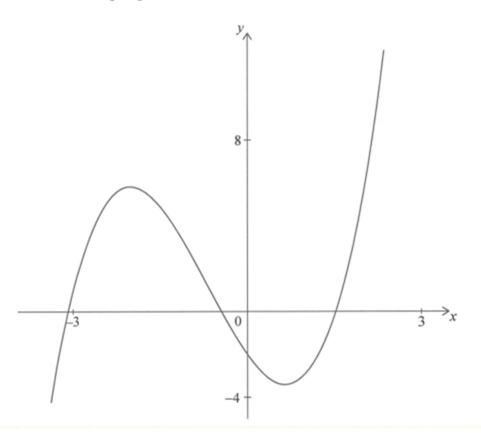
(c) Write down the range of h^{-1} .

[1]

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4. [Maximum mark: 5]

Consider the graph of the cubic function $f(x) = x^3 + 2x^2 - 4x - 2$. Part of the graph of y = f(x) is shown in the following diagram.



- (a) Write down the x-coordinate of
 - (i) the local maximum.
 - (ii) the local minimum.

[2]

(b) Hence, write down the interval where the function is decreasing.

[1]

The tangent to the curve at (1, -3) is parallel to the straight line y = 3x + 5.

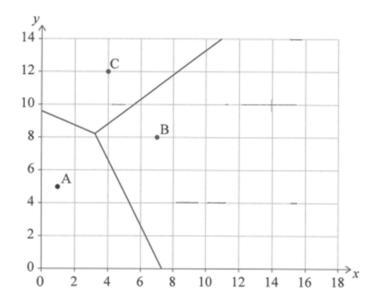
- (c) Write down
 - (i) the gradient of the tangent.
 - (ii) the equation of the tangent.

[2]

Turn over

5. [Maximum mark: 9]

A telecommunications company has identical cell towers in a rural area. They are located at the points A(1,5), B(7,8) and C(4,12). The coverage areas are divided as shown in the Voronoi diagram. All distances are in kilometres.



(a) Find the equation of the perpendicular bisector of [AB].

[4]

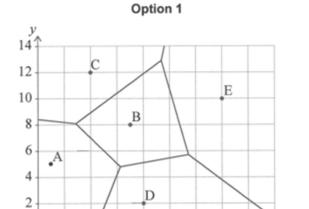
(Question 5 continued)

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The company is planning to improve the coverage of its cellular network in the area by adding two new towers. It identifies potential locations at the points D(8,2) and E(14,10).

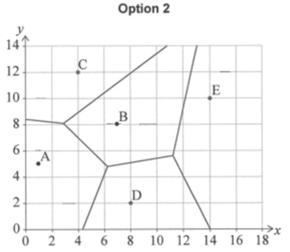
The company reviews the coverage areas and draws a new Voronoi diagram.

(b) Identify the correct Voronoi diagram from the options shown in the following diagrams.

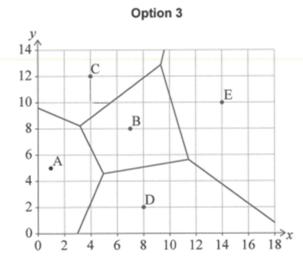


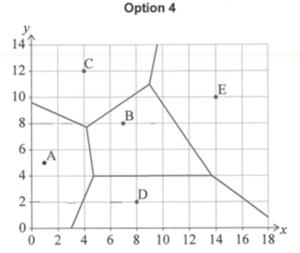
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12 14 16



[2]





(Question 5 continued)

Each tower provides guaranteed excellent coverage within a radius of $3\,\mathrm{km}$.

Pooja is at a beauty parlour located at the point (6, 4).

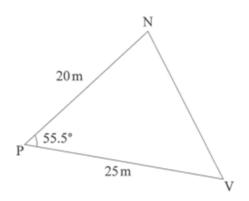
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Turn over

[Maximum mark: 9]

Three points N , P , and V are shown on the following diagram. NP is 20 metres, PV is 25 metres and $V\hat{P}N$ is 55.5° .

diagram not to scale



- (a) Find NV. [3]
- (b) Find \hat{PNV} . [3]
- (c) Hence or otherwise, find the shortest distance between P and [NV]. [3]

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7.	[Maximum]	mark	5
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The loudness of a sound, L, measured in decibels (dB) is determined by the intensity of the sound, I, measured in watts per square metre (Wm $^{-2}$). The relationship between loudness and intensity can be expressed using the logarithmic function

$$L = 10\log_{10}\left(\frac{I}{I_0}\right), I > 0$$

where I_0 is the reference intensity (the intensity of the least audible sound to the human ear).

The reference intensity I_0 is $10^{-12}\,\mathrm{Wm}^{-2}$.

The intensity of sound on a busy street is $10^{-5} \, \mathrm{Wm}^{-2}$.

(a) Calculate the loudness of the sound.

[2]

The sound of a jet engine reaches a loudness of $185\,\mathrm{dB}\,.$

(b) Determine the intensity of its sound. Give your answer in the form $a\times 10^k$ where $1\leq a\leq 10,\,k\in\mathbb{Z}$.

[3]

1025

[Maximum mark: 7]

Prakash is the leader of a customer service team and is interested in determining whether there is a relationship between a customer's satisfaction level and the type of service interaction they have experienced.

He collects data from a random sample of 250 customers and tracks their satisfaction level after three types of service interactions: in-person, online chat bots and website contact forms.

He categorizes the satisfaction levels as satisfied, neutral and dissatisfied.

He records the data in the following table.

		S	Satisfaction leve	el
		Satisfied	Neutral	Dissatisfied
	In-person	35	30	23
Type of service interaction	Online chat bots	31	39	23
	Website contact forms	19	28	22

Prakash performs a χ^2 test for independence at the 5% significance level.

The critical value is 9.488.

The null hypothesis, $H_{\rm 0}$, is the satisfaction level and the type of service interaction are independent.

- (a) State the alternative hypothesis for this test. [1]
- (b) Find the degrees of freedom for this test. [1]
- (c) Find χ^2_{colc} , the chi-squared test statistic. [2]

Prakash concludes that there is sufficient evidence to reject the null hypothesis.

- (d) (i) State whether Prakash is correct. Justify your answer.
 - (ii) Write down the conclusion for this test in context. [3]

(Question 8 continued)

Answers written on this page will not be marked.

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9.	[Maximum	mark	71
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Pascale owns a company that produces and sells curry powder. The rate of change of the company's profit, P, in Mauritian rupees (MUR) from producing x kilograms (kg) of curry powder is modelled by

$$\frac{dP}{dx} = -10x + 460, \ x \ge 0.$$

She makes a profit of $3300\,MUR$ when producing $10\,kg$ of curry powder.

(a) Find an expression for the company's profit, P, in terms of x. [5]

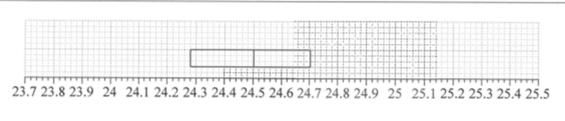
Pascale decides to increase the production of curry powder from $25\,\mathrm{kg}$ to $50\,\mathrm{kg}$.

(b) Find the increase in profit. [2]

10. [Maximum mark: 7]

The times, in seconds, for the fastest 16 women in a $50\,\mathrm{m}$ freestyle swimming championship event were recorded. All swimmers recorded different times.

Part of a box and whisker diagram for these times is shown in the following diagram.



Swimming times in seconds

(a) Write down the number of swimmers who took more than 24.70 seconds to complete the race.

[1]

(b) Find the interquartile range (IQR) for the data.

[2]

An outlier is defined as a value that satisfies one of the following:

- more than $1.5 \times IQR$ below the lower quartile
- more than 1.5 × IQR above the upper quartile.

Of the 16 women, the two fastest swimmers took 23.96 and 24.12 seconds and the two slowest women took 25.12 and 25.40 seconds to complete the race.

- (c) (i) Show that only one of these times is an outlier.
 - (ii) Complete the box and whisker diagram above.

[4]

11.	[Max	ximum	mark: 8]	
	com	pound	s out a loan for 50000 euros. She pays a nominal interest rate of 4.2% per year, led monthly. She must pay back the loan in 25 years through regular monthly, made at the end of each month.	
	(a)		the amount Sofia must pay back each month. Give your answer correct to decimal places.	[2]
			of eight years, Sofia wins the lottery and pays back the remainder of the loan early gle payment.	
	(b)	(i)	Find how much Sofia pays back in this single payment. Give your answer correct to two decimal places.	
		(ii)	Find the amount of money saved by Sofia by paying back the loan early.	[6]

Turn over

12. [Maximum mark: 7]

A team of researchers is using a model to predict the relative happiness of different countries. To do this, a value x is calculated based on easily measured parameters, for example, life expectancy, or available social support. It is assumed that higher values of x indicate greater happiness.

To test the model a survey is conducted in six countries, A, B, C, D, E and F. In these countries the level of happiness is assessed directly using questionnaires and given a score y, out of 10, with higher scores indicating greater happiness.

To select the countries for the survey, all countries are divided into three equal groups based on wealth and two countries are chosen randomly from each group.

(a) Write down the name of this type of sampling.

[1]

The results of the survey, along with the value obtained from the model, are given in the following table.

Country	'A	В	С	D	Е	F
Value from the model (x)	12.3	15.2	14.1	18.5	20.1	19.2
Happiness score (y)	5.2	7.3	6.2	6.9	8.0	7.2

The researchers will accept the model is a valid predictor of happiness score if the Pearson's product-moment correlation coefficient, r, is greater than 0.8.

- (b) (i) Find the value of r.
 - (ii) Hence state whether the model can be regarded as a valid predictor of happiness score.

[3]

(c) Find the equation of the regression line y on x.

[1]

For a particular country x = 17.2.

(d) Use the regression line to predict the happiness score for this country.

[2]

(Qu	estion 12 continued)